

Generation of the Coupling Circuit Parameters for the Coupled Oscillators Used in Antenna Arrays

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Abstract— This paper presents a new software which can generate in full-symbolic or numeric-symbolic form the Y , Z , H , and fundamental parameters of a two-port structure. Our procedure can also determine all the resonant frequencies of any two-port configuration as functions of the two-port circuit parameters. The procedure is based on the modified nodal equations in full-symbolic form. A new software called ANCSYANP (Analog Circuit Symbolic Analysis Program) was elaborated. This is an interactive tool that combines symbolic and numeric computational techniques, and which uses the facilities of the symbolic simulator Maple to manipulate the symbolic expressions. An illustrative example is done.

Keywords—component; symbolic analysis; two-port circuit; coupled oscillator; Y , Z , H parameters

I. INTRODUCTION

In the last years the coupled oscillators are used to control the phase in microwave antenna arrays. These devices produce oscillatory output signals of high frequency [1-9]. The radiation pattern of a phased antenna array is steered in a particular direction by controlling the phase gradient existing between the signals applied to adjacent elements of the array. The required inter-element phase shift can be obtained by detuning the free-running frequencies of the outermost oscillators in the array [2]. Furthermore, in [4] it is shown that the resulting inter-stage phase shift is independent of the number of oscillators in the array.

The aim of this paper is to present the symbolic analysis of the array of two coupled Van der Pol oscillators, considering the coupling circuit as a passive two-port circuit. Therefore, a new software which can generate in full-symbolic or numeric-symbolic form the Y , Z , H , and fundamental parameters of any two-port structure was developed. The procedure is based on the modified nodal equations of the entire circuit formulated in full-symbolic form.

The paper is organized as follows: a system of two Van der Pol oscillators coupled through a RLC circuit is presented in section II. In section III, the symbolic generation of the coupling circuit parameters with the new software is described followed by conclusions.

II. TWO COUPLED VAN DER POL OSCILLATORS

The theory of coupled microwave oscillators is the subject of increasing research activity. Simple Van der Pol oscillators, coupled through a resonant network that produces a constant magnitude and phase delay between the oscillators, provided a satisfactory model for a lot of applications [3].

Fig. 1,a illustrates a system of two Van der Pol oscillators coupled through a complex two-port circuit shown in Fig. 1,b.

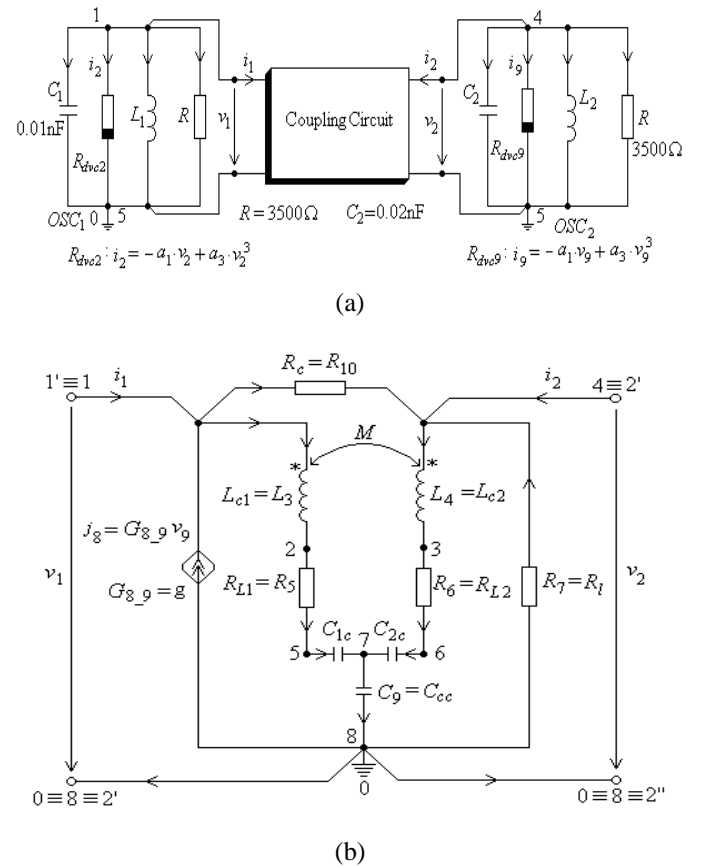


Figure 1. a) Two Van der Pol oscillators coupled through the circuit in Fig. 1, b) Structure of the coupling circuit.

The choice of this model is justified by its simplicity regarding the analytical calculations, as presented in [5, 6]. The active parts of the two Van der Pol oscillators are modeled by two voltage-controlled nonlinear resistors. The nonlinear characteristics of the two voltage-controlled nonlinear resistors are expressed as follows:

$$i_{nel1} = -av_1 + bv_1^3; \quad i_{nel2} = -av_2 + bv_2^3, \quad (1)$$

where a is the negative conductance necessary to start the oscillation and b a parameter used to model the saturation phenomenon.

Assuming a perfect oscillation so that $v(t) = A \cos(\omega_0 t)$ and according to (1), the expressions of the currents through the two nonlinear resistors can be written as:

$$i(t) = \left(-a + \frac{3}{4}bA^2 \right) A \cos(\omega_0 t) + b \frac{A^3}{4} \cos(3\omega_0 t) \quad (2)$$

Therefore, the two Van der Pol oscillators in Fig. 1, b can be modeled by a quasi-linear representation, replacing the two nonlinear resistors by the following conductances:

$$G_{02} = -a + \frac{3b}{4}A_1^2 \quad \text{and} \quad G_{09} = -a + \frac{3b}{4}A_2^2, \quad (3)$$

where A_1 and A_2 are the magnitudes of voltages v_1 and v_2 .

Performing a Spice simulation in transient behavior we get the magnitudes values $A_1 = 1.6032$ V, $A_2 = 2.4431$ V, and the synchronization frequency $f_s = 1.1512$ GHz, for the initial conditions $v_1(0) = 2.0$ V and $v_2(0) = 1.0$ V. Thus, in sinusoidal behavior the two voltage-controlled nonlinear resistors can be substituted by two linear resistors having negative slopes. In this case the circuit in Fig. 1 can be analyzed by the complex representation method [10, 11].

III. SYMBOLIC GENERATION OF THE COUPLING CIRCUIT PARAMETERS

The aim of this section is to generate in full-symbolic or numeric-symbolic form the Y , Z , H , and fundamental parameters of the coupling circuit of the two Van der Pol oscillators. To this end the coupling circuit is represented by a passive linear two-port circuit, as shown in Fig. 2.

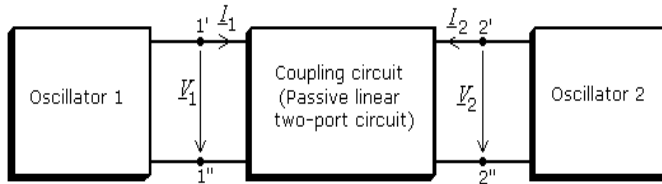


Figure 2. General scheme of two coupled oscillators.

In these conditions, the equations in complex admittances of the passive linear two-port are expressed as follows:

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \quad (4)$$

where the transfer admittances are defined in the expressions below:

$$\begin{aligned} Y_{11} &= \left. \frac{d I_1}{d V_1} \right|_{V_2=0} = Y_{ii}; & Y_{12} &= \left. \frac{d I_1}{d V_2} \right|_{V_1=0} = -Y_{io}; \\ Y_{21} &= \left. \frac{d I_2}{d V_1} \right|_{V_2=0} = Y_{oi}; & Y_{22} &= \left. \frac{d I_2}{d V_2} \right|_{V_1=0} = -Y_{oo}. \end{aligned} \quad (5)$$

Furthermore, the equations in complex impedance of a passive linear two-port can be written as:

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \quad (6)$$

where the transfer impedances are defined as follows:

$$\begin{aligned} Z_{11} &= \left. \frac{d V_1}{d I_1} \right|_{I_2=0} = Z_{ii}; & Z_{12} &= \left. \frac{d V_1}{d I_2} \right|_{I_1=0} = -Z_{io}; \\ Z_{21} &= \left. \frac{d V_2}{d I_1} \right|_{I_2=0} = Z_{oi}; & Z_{22} &= \left. \frac{d V_2}{d I_2} \right|_{I_1=0} = -Z_{oo}. \end{aligned} \quad (7)$$

In H parameters the equations are:

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} \quad (8)$$

and the definitions of the parameters are:

$$\begin{aligned} H_{11} &= \left. \frac{d V_1}{d I_1} \right|_{V_2=0} = Z_{ii}; & H_{12} &= \left. \frac{d V_1}{d V_2} \right|_{I_1=0} = A_{io}; \\ H_{21} &= \left. \frac{d I_2}{d I_1} \right|_{V_2=0} = B_{oi}; & H_{22} &= \left. \frac{d I_2}{d V_2} \right|_{I_1=0} = -Y_{oo}. \end{aligned} \quad (9)$$

If the fundamental (transfer) complex parameters are of interest, the equations have the form:

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} \quad (10)$$

and the parameters are defined as:

$$\begin{aligned} A &= \left. \frac{d V_1}{d V_2} \right|_{I_2=0} = \frac{1}{A_{oi}}; & B &= \left. \frac{d V_1}{d I_2} \right|_{V_2=0} = \frac{1}{Y_{oi}}; \\ C &= \left. \frac{d I_1}{d V_2} \right|_{I_2=0} = \frac{1}{Z_{oi}}; & D &= \left. \frac{d I_1}{d I_2} \right|_{V_2=0} = \frac{1}{B_{oi}}. \end{aligned} \quad (11)$$

In order to automatically generate (in symbolic or numeric-symbolic form) all the above parameters associated to the two-port used to model the coupling circuit, we adapted the general software - ANCSYANP (Analog Circuit Symbolic Analysis Program) [10, 11]. The new analysis tool based on the modified nodal analysis (MNA) generates, starting from the circuit netlist, the Y , Z , H , and fundamental parameters, for any linear and/or nonlinear (in any driving point) time-invariant two-port analog circuit, in symbolic form. The excess elements are also taken into account. It is an interactive tool that combines symbolic and numeric computational techniques, and which uses the facilities of symbolic simulator Maple to manipulate the symbolic expressions.

Let us now remember the equations in Laplace domain corresponding to the MNA:

$$\begin{bmatrix} \mathbf{Y}_{n-1,n-1}(s) & \mathbf{B}_{n-1,m}(s) \\ \mathbf{A}_{m,n-1}(s) & \mathbf{Z}_{m,m}(s) \end{bmatrix} \cdot \begin{bmatrix} \mathbf{V}_{n-1}(s) \\ \mathbf{I}_m(s) \end{bmatrix} = \begin{bmatrix} \mathbf{I}_{sc,n-1}(s) \\ \mathbf{E}_m(s) \end{bmatrix}, \quad (12)$$

where: $\mathbf{Y}_{n-1,n-1}(s)$ is the operational admittance matrix of size $(n-1) \times (n-1)$; $\mathbf{B}_{n-1,m}(s)$ represents a matrix of size $(n-1) \times m$ with entries -1, 0, 1 and the current transfer factor (gain) of the current-controlled current sources; $\mathbf{A}_{m,n-1}(s)$ is a $m \times (n-1)$ matrix with entries -1, 0, 1 and the voltage transfer factor (gain) of the voltage-controlled voltage sources; $\mathbf{Z}_{m,m}(s)$ represents a square matrix of $m \times m$ size which contains the operational trans-impedances of the current-controlled voltage sources and the operational inductor impedances with negative sign; $\mathbf{I}_{sc,n-1}(s)$ is the operational short-circuit current vector injected in $n-1$ independent nodes (including the ones resulted from the independent sources which simulate the initial conditions) and $\mathbf{E}_m(s)$ represents the vector of the Laplace transforms (with negative sign) of the electromotive forces corresponding to the ideal independent voltage sources and of the electromotive forces corresponding to the ideal independent voltage sources which simulate the inductor initial conditions. The unknown vector contains the Laplace transform vector of the electrical potentials corresponding to the $n-1$ independent circuit nodes, $\mathbf{V}_{n-1}(s)$, and the Laplace transform vector of the branch currents non-compatible with the nodal analysis, $\mathbf{I}_m(s)$.

Based on the definition of the circuit parameters we apply suitable ideal independent sources to the input ports of the circuit in Fig. 2. Describing the circuit behavior by MNM the program computes in symbolic form the desired parameters.

ANCSYANP has the following capabilities:

- Automatic generation in full symbolic, partially symbolic and/or numeric form of the modified nodal equations (MNE) for the linear and/or nonlinear circuits;
- Full symbolic, partially symbolic and/or numeric computation of all branch currents and voltages;
- Computation of the numeric value of the system matrix determinant;
- Computation of the poles and zeros and representation of the natural frequencies in the complex plane;

- Generation in full symbolic, partially symbolic or numeric form of the two-port parameters;
- Computation of the sensitivities with respect to any circuit parameter and the 3D representation;

In general, Y parameters are important in the analysis of two oscillators coupled through a passive linear two-port circuit. For the circuit in Fig. 1, a, the equations in complex form, when the two voltage-controlled nonlinear resistors are substituted by two linear resistors according to (3), can be written as:

$$\begin{aligned} \underline{Y}_1 \underline{V}_1 &= -\underline{Y}_{11} \underline{V}_1 - \underline{Y}_{12} \underline{V}_2 \\ \underline{Y}_2 \underline{V}_2 &= -\underline{Y}_{21} \underline{V}_1 - \underline{Y}_{22} \underline{V}_2, \end{aligned} \quad (13)$$

where

$$\begin{aligned} \underline{Y}_1 &= -a + \frac{3}{4} b A_1^2 + j\omega C_1 + \frac{1}{j\omega L_1} + \frac{1}{R} \\ \underline{Y}_2 &= -a + \frac{3}{4} b A_2^2 + j\omega C_2 + \frac{1}{j\omega L_2} + \frac{1}{R} \end{aligned} \quad (14)$$

Running ANCSYANP all transfer admittances in full symbolic form are obtained. Because the expressions of these admittances are too large, we shall present only \underline{Y}_{11} expression:

$$\begin{aligned} \underline{Y}_{11} &= (C_c^2 C_{cc} (L_c^2 - M^2) \omega^4 - j C_c^2 L_c C_{cc} (2R_L + R_c) \omega^3 + \\ & (-2L_c C_c^2 + 2C_c^2 M - 2L_c C_c C_{cc} + g L_c C_c^2 R_c - g M C_c^2 R_c - C_c^2 C_{cc} R_L^2 - C_{cc} C_c^2 R_c R_L) \omega^2 + \\ & j(-g C_c^2 R_c R_L + 2C_c C_{cc} R_L + C_c^2 R_c + 2C_c^2 R_L + C_{cc} C_c R_c) \omega + 2C_c + C_{cc} - g C_c R_c) / \\ & (R_c (C_c^2 C_{cc} (L_c^2 - M^2) \omega^4 - 2j L_c C_c^2 C_{cc} R_L \omega^3 + (-2L_c C_c C_{cc} - C_c^2 C_{cc} R_L^2 - 2L_c C_c^2 + \\ & 2C_c^2 M) \omega^2 + 2j C_c R_L (C_{cc} + C_c) \omega + 2C_c + C_{cc})) \end{aligned} \quad (15)$$

If we denote $\underline{X} = \frac{\underline{V}_1}{\underline{V}_2} = \frac{A_1}{A_2} e^{j\phi_{21}}$ and take the values of the circuit parameters from Fig. 1, with $a = 0.0085$, $b = 0.00071$, $A_1 = 1.6032$ V, and $A_2 = 2.4431$ V, then (13) becomes a system of two equations with two unknowns, \underline{X} and ω , so that:

$$\begin{aligned} \underline{X}_{analytic} &= 0.51312 + 0.38539 j \\ X_{analytic} &= 0.6563 ; \omega_{analytic} = 7.234 \cdot 10^9 \text{ [rad/s]} \\ \Delta\phi_{21_analytic} &= \arg(\underline{X}_{analytic}) = 35.97^\circ. \end{aligned} \quad (16)$$

Performing a Spice simulation we get:

$$\begin{aligned} X_{Spice} &= 0.6566 ; \Delta\phi_{21_Spice} = 36.0^\circ \\ \omega_{Spice} &= 7.2295 \cdot 10^9 \text{ [rad/s]}. \end{aligned} \quad (17)$$

We can remark the good agreement between these results.

In order to determine the transfer impedances (Z parameters) for a two-port passive linear circuit, we connected to the input port (0 – 1) and to the output port (0 – 4) the ideal independent current source \underline{I}_1 ($J_1(s)$) and \underline{I}_2 ($J_2(s)$), respectively, with arbitrary current values, as it shows in Fig. 3.

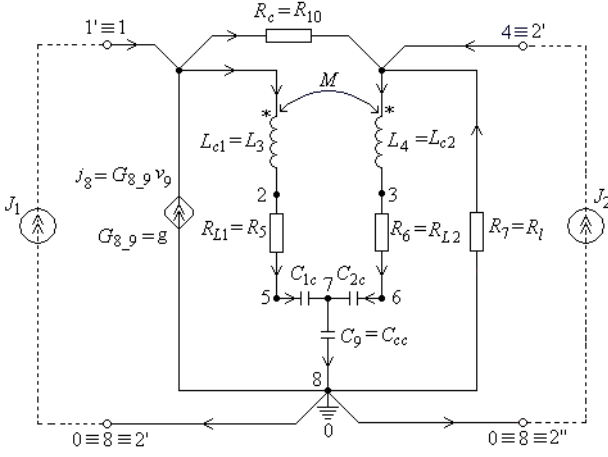


Figure 3. The equivalent two-port circuit for Z parameter computing

According to the Fig. 2, the transfer impedances are computed with the following relations:

$$\begin{aligned} Z_{11}(s) &= \left. \frac{V_1(s)}{J_1(s)} \right|_{J_2(s)=0} ; Z_{12}(s) = \left. \frac{V_1(s)}{J_2(s)} \right|_{J_1(s)=0} ; \\ Z_{21}(s) &= \left. \frac{V_4(s)}{J_1(s)} \right|_{J_2(s)=0} ; Z_{22}(s) = \left. \frac{V_4(s)}{J_2(s)} \right|_{J_1(s)=0} . \end{aligned} \quad (18)$$

Running ANCSYANP all transfer impedances in full symbolic form are obtained. Because the expressions of these impedances are too large, we shall present only Z_{12} expression:

$$\begin{aligned} Z_{12} = & (C^2 C_c R_l (M^2 - L^2) \omega^4 + j C^2 C_c R_l (M R_c + 2 L R_L) \omega^3 + \\ & (2 C^2 L R_L - 2 C^2 M R_l + 2 L C C_c R_l + C^2 C_c R_L^2 R_l + g C^2 L R_l R_c - g C^2 M R_l R_c) \omega^2 + \\ & j (-2 C^2 R_l R_l - C^2 R_l R_c - 2 C C_c R_l R_L - C^2 g R_L R_l R_c) \omega - 2 C R_l - C_c R_l - C g R_l R_c) / \\ & (C^2 C_c (L^2 - M^2) \omega^4 + j (-2 L C^2 C_c R_l - L C^2 C_c R_c + 2 C^2 M C_c R_l - L C^2 C_c R_L) \omega^3 + \\ & (-C^2 C_c R_l R_c + 2 C^2 L g R_l - 2 C^2 M g R_l - 2 L C^2 + 2 C^2 M - 2 L C C_c - C^2 C_c R_L R_c - \\ & 2 C^2 C_c R_L R_l - C^2 C_c R_L^2 + C^2 L g R_c - C^2 M g R_c) \omega^2 + j (-2 C^2 g R_L R_l + 2 C C_c R_l - \\ & C^2 g R_l R_c - C^2 g R_L R_c + C^2 R_c + 2 C^2 R_L + C C_c R_c + 2 C C_c R_L) \omega + 2 C + C_c - \\ & C g R_c - 2 C g R_l) \end{aligned} \quad (19)$$

The resonant frequencies corresponding to the four transfer impedances have the following expression:

$$\begin{aligned} f_{r_{-Z_{11}}} &= 0.9207 \text{ GHz}; f_{r_{-Z_{12}}} = 1.019 \text{ GHz}; \\ f_{r_{-Z_{21}}} &= 1.0188 \text{ GHz}; f_{r_{-Z_{22}}} = 0.92058 \text{ GHz}. \end{aligned} \quad (20)$$

We denote that the resonant frequencies for Z_{11} and Z_{22} (Z_{12} and Z_{21}) transfer impedances are very closed.

The resonant frequencies for the four transfer admittances have the following expression:

$$\begin{aligned} f_{r1Y_{11}} &= 0.859 \text{ GHz}; f_{r2Y_{11}} = 0.92 \text{ GHz}; f_{r3Y_{11}} = 0.947 \text{ GHz}; \\ f_{r1Y_{22}} &= 0.859 \text{ GHz}; f_{r2Y_{22}} = 0.92 \text{ GHz}; f_{r3Y_{22}} = 0.947 \text{ GHz}; \\ f_{r1Y_{12}} &= 0.856 \text{ GHz}; f_{rY_{12}} = 0.95 \text{ GHz}; f_{r3Y_{12}} = 1.025 \text{ GHz}; \\ f_{r1Y_{21}} &= 0.856 \text{ GHz}; f_{rY_{21}} = 0.95 \text{ GHz}; f_{r3Y_{21}} = 1.025 \text{ GHz}. \end{aligned} \quad (21)$$

We denote that the resonant frequencies corresponding to the Y_{11} and Y_{22} (Y_{12} and Y_{21}) transfer admittances are identical.

IV. CONCLUSIONS

Using a suitable software we performed the symbolic analysis of an antenna array in order to compute in symbolic form the coupling circuit parameters.

The existing software was enhanced with dedicated routines for generating Y (admittance), Z (impedance), H and fundamental parameters of the coupling network, which is modeled by a passive linear two-port circuit.

The procedure developed can determine all the resonant frequencies of any two-port configuration as functions of the two-port circuit parameters.

The symbolic expressions of the coupling circuit parameters are useful in writing the equations in the sinusoidal behavior (when the nonlinear resistors can be substituted by the linear resistors) of an array of coupled oscillators for the automatic design of these devices.

The program can be used also for a complete analysis in order to compute the phase shift and the synchronous frequency of the antenna array.

The values of the phase shift and the synchronous frequency (analytic expressions) obtained by this approach are in a good agreement with the values obtained by Spice simulation.

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